

6.1 Imaginary Numbers

PRACTICE

Simplify each of the following powers of i into either -1 , 1 , i , or $-i$.

1. $i^2 = -1$

2. $i^9 = i^1 = i$
$$\begin{array}{r} 4 \overline{) 9} \\ \underline{8} \\ 1 \\ \underline{0} \\ 0 \end{array}$$

3. $i^{19} = i^3 = -i$
$$\begin{array}{r} 4 \overline{) 19} \\ \underline{16} \\ 3 \\ \underline{0} \\ 0 \end{array}$$

4. $i^{42} = i^2 = -1$
$$\begin{array}{r} 4 \overline{) 42} \\ \underline{40} \\ 2 \\ \underline{0} \\ 0 \end{array}$$

5. $i^{80} = i^4 = 1$
$$\begin{array}{r} 4 \overline{) 80} \\ \underline{80} \\ 0 \\ \underline{0} \\ 0 \end{array}$$

Simplify the following square roots.

6. $\sqrt{45}$
 $\sqrt{9} \cdot \sqrt{5}$
 $3\sqrt{5}$

7. $\sqrt{-20}$
 $i\sqrt{20}$
 $i\sqrt{4} \cdot \sqrt{5}$
 $2i\sqrt{5}$

8. $\sqrt{-75}$
 $i\sqrt{75}$
 $i\sqrt{25} \cdot \sqrt{3}$
 $5i\sqrt{3}$

9. $-\sqrt{27}$
 $-\sqrt{9} \cdot \sqrt{3}$
 $-3\sqrt{3}$

10. $\sqrt{-64}$
 $i\sqrt{64}$
 $8i$

Simplify the following. Identify as Real or Imaginary.

11. $\sqrt{-4} + \sqrt{-9}$
 $2i + 3i$
 $5i$

Real

Imaginary

12. $(2i)^2$
 $(2i)(2i)$
 $4i^2$
 $4(-1) = -4$

Real

Imaginary

13. $2i(4i)$
 $8i^2$
 $8(-1)$
 -8

Real

Imaginary

14. $4 + 3i^2$
 $4 + 3(-1)$
 $4 - 3$
 1

Real

Imaginary

15. $2\sqrt{-40}$
 $2i\sqrt{40}$
 $2i\sqrt{4} \cdot \sqrt{10}$
 $2i \cdot 2 \cdot \sqrt{10}$
 $4i\sqrt{10}$

Real

Imaginary

16. $\frac{6i}{2}$
 $\frac{36i}{12} = 3i$

Real

Imaginary

17. $2i^2(3i^2) + 2$
 $6i^4 + 2$
 $6(1) + 2$
 $6 + 2$
 8

Real

Imaginary

18. $4i + \sqrt{-1}$
 $4i + i$
 $5i$

Real

Imaginary

19. $5i^{16} + 3i^{22} + i^{26}$
 $5i^4 + 3i^2 + i^2$
 $5(1) + 3(-1) + (-1)$
 $5 - 3 - 1$
 1

Real

Imaginary

Solve. Express your radical solutions in the simplest form.

20. $x^2 + 2 = -26$

$$\sqrt{x^2} = \sqrt{-28}$$

$$x = \pm i\sqrt{28}$$

$$x = \pm i\sqrt{4 \cdot 7}$$

$$x = \pm 2i\sqrt{7}$$

21. $9 - 4y^2 = 57$

$$-4y^2 = 48$$

$$\sqrt{y^2} = \sqrt{-12}$$

$$y = \pm i\sqrt{12}$$

$$y = \pm i\sqrt{4 \cdot 3}$$

$$y = \pm 2i\sqrt{3}$$

22. $2m^2 = -200$

$$\sqrt{m^2} = \sqrt{-100}$$

$$m = \pm 10i$$

Solve. Express your radical solutions in the simplest form.

23. $(x-3)^2 + 28 = 4$

$$\sqrt{(x-3)^2} = \sqrt{-24}$$

$$x-3 = \pm i\sqrt{24}$$

$$x = 3 \pm i\sqrt{24}$$

$$x = 3 \pm i\sqrt{4 \cdot 6}$$

$$x = 3 \pm 2i\sqrt{6}$$

24. $-60 = 2(h+7)^2 + 4$

$$\frac{-64}{2} = \frac{2(h+7)^2}{2}$$

$$\sqrt{-32} = \sqrt{(h+7)^2}$$

$$\pm i\sqrt{32} = h+7$$

$$-7 \pm i\sqrt{32} = h$$

$$-7 \pm i\sqrt{16 \cdot 2} = h$$

$$-7 \pm 4i\sqrt{2} = h$$

25. $\frac{3(t-6)^2}{3} = \frac{-75}{3}$

$$\sqrt{(t-6)^2} = \sqrt{-25}$$

$$t-6 = \pm i\sqrt{25}$$

$$t = 6 \pm i\sqrt{25}$$

$$t = 6 \pm 5i$$

26. $p^2 + 70 = -2p^2 + 4$

$$3p^2 + 70 = 4$$

$$\frac{3p^2}{3} = \frac{-66}{3}$$

$$\sqrt{p^2} = \sqrt{-22}$$

$$p = \pm i\sqrt{22}$$

27. $(v-3)^2 - 5 = 0$

$$\sqrt{(v-3)^2} = \sqrt{5}$$

$$v-3 = \sqrt{5}$$

$$v = 3 + \sqrt{5}$$

28. $w^2 + 58 = 4$

$$\sqrt{w^2} = \sqrt{-54}$$

$$w = \pm i\sqrt{54}$$

$$w = \pm i\sqrt{9 \cdot 6}$$

$$w = \pm 3i\sqrt{6}$$

Perform the indicated operation.

Polynomials

29. $(2x^3 - 3x) + (4x^3 + 2x - 1)$

$$2x^3 - 3x + 4x^3 + 2x - 1$$

$$6x^3 - 1x - 1$$

Rational Expression

30. $\frac{x(3x)}{x(x+1)} + \frac{4(x+1)}{x(x+1)}$

$$\frac{3x^2 + 4x + 4}{x(x+1)}$$

Radicals

31. $\sqrt{8} + 4\sqrt{2}$

$$\sqrt{4 \cdot 2} + 4\sqrt{2}$$

$$2\sqrt{2} + 4\sqrt{2}$$

$$6\sqrt{2}$$