7.4 Exponential Modeling

NOTES

ALGEBRA 2

Write your questions here!

Financial Exponential Modeling!

______ is the amount of money after __ years.

__ is the principal (original amount of money).

___ is the interest rate (written as a decimal).

_____ is the number of times the interest is compounded (paid) per year.

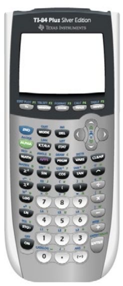
$e \approx$ ____________________

Periodic Compounding Interest:

Continuous Compounding Interest:

% increase / decrease:

1. You deposit $1,000 into an account that pays 6% interest compounded continuously. Write a model and then find the balance after 5 years.



2. You deposit $2,000 into an account that pays 4.8% interest compounded quarterly. Write a model and then find the balance after 10 years.

3. You have a home worth $100,000 and it is increasing in value at 2.5% per year. Write a model and then find the value of the home after 20 years.

WHAT IS e, the natural base?

Consider the expression $\left(1+\frac{1}{n}\right)^n$. As you increase the value of n, this expression will get closer and closer to the value of e. In other words, As $n \longrightarrow \infty$, $\left(1+\frac{1}{n}\right)^n \longrightarrow e$

n	1	10	100	100,000	1,000,000
$\left(1+\frac{1}{n}\right)^n$	**2**	**2.59374**	**2.70481**	**2.71827**	**2.71828**

$e \approx$

Simplify: e is a number, but the same rules for variables and exponents still apply.

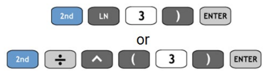
4. $e^{2x} \cdot e^{5x-1}$

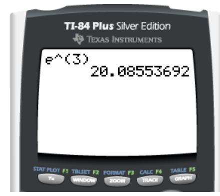
5. $5(e^{5-2x})^2$

6. $\frac{10e^x}{5e^{x-3}}$

Using a Calculator

7. $e^3 \approx$





8. $e^{-0.2} \approx$

Other Modeling Fun!

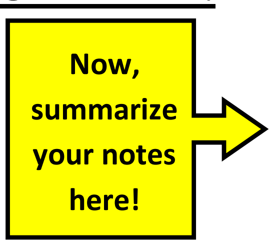
9. The population of the country New Beanland was 40 million in the year 2010 and has grown continuously in the years following. The population, P, in millions, of the country t years after 2010 can be modeled by the function $P(t) = 40e^{0.027t}$, where $t > 0$. What is the ***average rate of change*** of the population of New Beanland from 2010 to 2015. Give your answer to the nearest hundredth.

10. A scientist places 7.35 grams of a radioactive element with a half-life of 2 days in a dish. A d days, the number of grams of the element is given by the function $R(d) = 7.35\left(\frac{1}{2}\right)^{\frac{d}{2}}$. Which of the following are true?

Select all that apply:

- ☐ A. An approximate equivalent equation is $R(d) = 7.35(0.250)^d$.
- ☐ B. An approximate equivalent equation is $R(d) = 7.35(0.707)^d$.
- ☐ C. The base of this form of the equation can be interpreted to mean the element decays by 0.250 grams per day.
- ☐ D. The base of this form of the equation can be interpreted to mean the element decays by 0.707 grams per day.
- ☐ E. The base of this form of the equation can be interpreted to mean that about 25% of the element remains from one day to the next.
- ☐ F. The base of this form of the equation can be interpreted to mean that about 70.7% of the element remains from one day to the next.

SUMMARY:



7.4 Exponential Modeling

PRACTICE

For 1-8, use exponent properties to simplify. Your answer should contain only positive exponents

1. $e^3 \cdot e^{-5}$
2. $-\frac{e^x}{2e}$
3. $\frac{5e^x}{e^{5x}}$
4. $(2e^{-4x})^3$
5. $\frac{e^{6x-1}}{e^{x-2}}$
6. $\frac{e^{5x}}{e^3}$
7. $(5e^{2+3x})^2$
8. $(-3e^{6x})^3$

For 9-12, use a calculator to evaluate the expression. Round the result to three decimal places.

9. $4e^2$
10. $-10e^{-2}$
11. $52e^{-4}$
12. $-4e^3$

For 13 – 18, use one of the three generic models to help you create a specific model for each compounding interest scenario. Then, use your model to calculate the balance for the given amount of time.

Compounding Interest (continuous compounding)	Compounding Interest (periodic compounding)	% increase/decrease per unit of time
$A = Pe^{rt}$	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	$f(x) = ab^x$

13. You deposit $800 in an account that pays 5.7% annual interest compounded continuously. How much will you have after 13 years?

14. Your home is worth $200,000 and increases in value by 2.5% per year. How much will it be worth in 20 years?

15. You deposit $5 in an account that pays 24% annual interest compounded monthly. How much will you have after 20 years?

16. You deposit $468 into a mutual fund account. It decreases in value by 1% per week for six months. How much do you have after 14 weeks?

17. You deposit $1000 in an account that pays 2% annual interest compounded quarterly. How much will you have after 8 years?

18. You deposit $3500 in an account that pays 8.2% annual interest compounded continuously. How much will you have after 2 years?

19. A bank pays depositors 2% interest rate compounded semiannually. Let P represent an initial deposit let t represent the number of years that the deposit is in the bank. The expression $P\left(1+\frac{0.02}{2}\right)^{2t}$ can be used to determine the account balance after t years. Which expression accurate reflects the annual interest rate?

- ☐ A. $P(1.01)^t$
- ☐ C. $P(1.0201)^t$
- ☐ B. $P(1.21)^t$
- ☐ D. $P(1.0404)^t$

20. a. Assuming the annual inflation rate has been about 1.6% in the last 100 years, how much does a gold chain that cost Sully $20 in 1990, cost today?

b. What would a gold chain bought today for $20 cost in 1990? ← *Think negative years!*

21. The Algebros added Common Core courses to their website flippedmath.com. The following data represents the number of hits on the Algebra 2 common core site since it went online:

x	1	2	3	4	5	6	7	8	9	10
y	22	39	70	126	227	408	735	1322	2380	4285

a. Use your calculator to find an exponential model for the data. If you need help, refer to the calculator help video in the 7.3 section under the main lesson.

b. According to your model, about how many hits were there in the 12th month?

c. How many hits would there be in 3 years (36th month)? What is wrong with this number?

22. Functions of the form $P(t) = P_o e^{-rt}$ describe exponential decay, where r is the decay rate, P_o is the initial amount and t is time.

a. Suppose a certain radioactive element has an annual decay rate of 10%. Starting with a 200 gram sample of the element, how many grams will be left in 3 years?

b. Suppose a 50 gram sample of the element in ***part a*** was found. How much of the element existed 5 years ago?

REVIEW…..Solve each equation. Check for extraneous solutions.

22.

$$1 = \frac{5}{6x-5}$$

23.

$$\frac{3}{a+5} = \frac{2}{a^2+5a} + \frac{1}{a+5}$$

7.4 Exponential Modeling

WRAP UP

1. You deposit $25 in an account that pays 14% annual interest compounded weekly. How much will you have after 20 years?

2. Suppose the amount above was invested with continuously compounded interest. How much would you then have after 20 years?

EXIT TICKET –

A biologist studying a population of alligators in a State Park determines that the population can be modelled by the formula $f(t) = 120{,}000(1.015)^t$, where $f(t)$ represents the population after t years. An intern studying with the biologist makes this claim: "Based on the formula, after 1 year the population will have increased by 1800. Since 1800 divided by 12 is 150, we can used the fact that the population increases by 150 Alligators per month to predict the future population of alligators in the park."

Explain why the intern's claim is not valid.

Modify the initial given formula such that it represents the predicted population after m months, and use the formula to predict the population after 50 months.