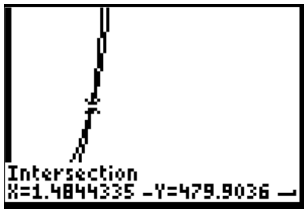
8.4 Solving Exponential Equations

PRACTICE

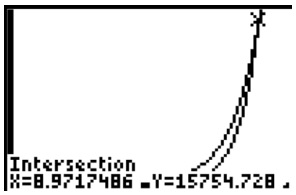
Directions: Solve the equation with a graphing calculator. Round to the nearest thousandth.

1) $4^{3x} = 12^{x+1}$



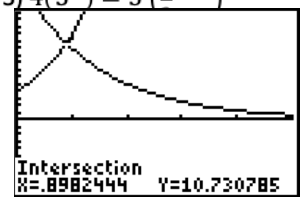
$X = 1.484$

2) $2e^{x} = 4^{x-2}$



$X = 8.972$

3) $4(3^{x}) = 5\left(\frac{1}{2}^{x-2}\right)$



$X = 0.898$

Directions: Solve the equation. Give the EXACT and APPROXIMATE (to nearest thousandth) answers.

4) $5^{x-2} = 24$

$\log_5 5^{x-2} = \log_5 24$

$x - 2 = \log_5 24$

$+2 \quad +2$

$x = \log_5 24 + 2$

$x \approx 3.975$

5) $74 = 8^{2x}$

$\log_8 74 = \log_8 8^{2x}$

$\log_8 74 = 2x$

$\frac{\log_8 74}{2} = \frac{2x}{2}$

$\frac{\log_8 74}{2} = x$

$x \approx 1.035$

6) $8^{-5a} - 5 = 53$

$+5 \quad +5$

$8^{-5a} = 58$

$\log_8 8^{-5a} = \log_8 58$

$\frac{-5a}{-5} = \frac{\log_8 58}{-5}$

$a = \frac{\log_8 58}{-5}$

$a \approx -0.391$

7) $-6e^{n-3} - 4 = -24$

$+4 \quad +4$

$\frac{-6e^{n-3}}{-6} = \frac{-20}{-6}$

$e^{n-3} = \frac{10}{3}$

$\ln e^{n-3} = \ln\left(\frac{10}{3}\right)$

$n - 3 = \ln\left(\frac{10}{3}\right)$

$n = \ln\left(\frac{10}{3}\right) + 3$

$n \approx 4.204$

8) $2(3^{x+2}) - 10 = 32$

$+10 \quad +10$

$\frac{2(3^{x+2})}{2} = \frac{42}{2}$

$3^{x+2} = 21$

$\log_3 3^{x+2} = \log_3 21$

$x + 2 = \log_3 21$

$-2 \quad -2$

$x = \log_3 21 - 2$

$x \approx 0.771$

9) $2(4^{x}) = 4^{2x}$

$\log_4(2 \cdot 4^{x}) = \log_4 4^{2x}$

$\log_4 2 + \log_4 4^{x} = 2x$

$\log_4 2 + x = 2x$

$-x \quad -x$

$\log_4 2 = x$

$x = 0.5$

10) $\frac{15(2^{x+2})}{3} = \frac{3(2^{3x})}{3}$

$5(2^{x+2}) = 2^{3x}$

$\log_2(5 \times 2^{x+2}) = \log_2 2^{3x}$

$\log_2 5 + \log_2 2^{x+2} = 3x$

$\log_2 5 + x + 2 = 3x$ (−x)

$\frac{\log_2 5 + 2}{2} = \frac{2x}{2}$

$\frac{\log_2 512}{2} = x$

$x \approx 2.161$

11) $\frac{4e^{3x-1}}{4} = \frac{20e^{x}}{4}$

$e^{3x-1} = 5e^{x}$

$\ln e^{3x-1} = \ln(5e^{x})$

$3x - 1 = \ln 5 + \ln e^{x}$

$3x - 1 = \ln 5 + x$ (−x)

$2x - 1 = \ln 5$

$\frac{2x}{2} = \frac{\ln 5 + 1}{2}$

$x = \frac{\ln 5 + 1}{2}$

$x \approx 1.305$

Compounding Interest (continuous compounding)	Compounding Interest (periodic compounding)	% increase/decrease per unit of time
$A = Pe^{rt}$	$A = P\left(1 + \frac{r}{n}\right)^{nt}$	$f(x) = ab^{x}$

12) Mr. Brust invests $5000 in an account that is compounded monthly at a rate of 9%. How many years will it take him to have $23,000 in the account? $n = 12$ $r = .09$

$\frac{23,000}{5000} = \frac{5000(1 + \frac{.09}{12})^{12t}}{5000}$

$\frac{23}{5} = (1.0075)^{12t}$

$\log_{1.0075}\left(\frac{23}{5}\right) = 12t$

$\frac{204.236}{12} = \frac{12t}{12}$

$t \approx 17.02$ years

13) Mr. Bean puts $5000 in a mutual fund that increases in value by 11% each year. How many years will it take him to reach $23,000 in his mutual fund?

$\frac{23,000}{5000} = \frac{5000(1.11)^{x}}{5000}$

$\frac{23}{5} = 1.11^{x}$

$\log_{1.11}\left(\frac{23}{5}\right) = x$

$14.62 \text{ yrs} = x$

14) Mr. Kelly puts $5000 into an account that is compounded continuously at a rate of 10%. How long will it take for him to have $23,000 in the account? $r = .10$

$\frac{23000}{5000} = \frac{5000e^{.10t}}{5000}$

$\frac{23}{5} = e^{.10t}$

$\ln\left(\frac{23}{5}\right) = \ln e^{.10t}$

$\frac{1.526}{.1} = \frac{.10t}{.1}$

$15.26 \text{ yrs} = t$

15) At his wedding Mr. Kelly had a statue of the Algebros made of ice. He estimated that every 4 days half of the weight of the ice sculpture has melted. How many days until 10% of the sculpture is left? $a = 100\%$ $y = 10\%$

$y = a \cdot b^{x}$

$10 = 100\left(\frac{1}{2}\right)^{x/4}$

$.1 = \frac{1}{2}^{x/4}$

$\log_{.5} .1 = \log_{.5} \frac{1}{2}^{x/4}$

$3.32 = \frac{x}{4}$

$13.29 \text{ days} = x$

16) Mr. Bean collects crickets. He has ten crickets currently and realizes the population is tripling every 2 weeks. How long will it take for his cricket population to reach 500? $a = 10$ $b = 3$ $y = 500$

$y = a \cdot b^{x}$

$\frac{500}{10} = \frac{10}{10}(3)^{x/2}$

$50 = 3^{x/2}$

$\log_3 50 = \log_3 3^{x/2}$

$\log_3 50 = x/2$

$3.56 = x/2$

$x = 7.12$ weeks