

3.4 Solve Rational Equations

PRACTICE

Solve each equation. Check for extraneous solutions.

1. $2a \left(\frac{a+4}{2a} + 2 = \frac{3}{2} \right) = \frac{2a(a+4)}{2a} + \frac{2a \cdot 2}{1} = \frac{2a \cdot 3}{2}$

$a \neq 0$

$$a+4+4a=3a$$

$$5a+4=3a$$

$$\begin{array}{r} -5a \\ -5a \end{array}$$

$$\frac{4}{-2} = \frac{-2a}{-2}$$

$$\boxed{-2=a}$$

2. $m^2 \left(\frac{3m+15}{m^2} - \frac{1}{m} = \frac{1}{1} \right) = \frac{m^2(3m+15)}{m^2} - \frac{m^2 \cdot 1}{m} = \frac{1 \cdot m^2}{1}$

$m \neq 0$

$$3m+15-m=m^2$$

$$2m+15=m^2$$

$$\begin{array}{r} -2m \\ -2m \end{array}$$

$$15=m^2-2m$$

$$\begin{array}{r} -15 \\ -15 \end{array}$$

$$0=m^2-2m-15$$

$$0=(m-5)(m+3)$$

$\boxed{x=5, -3}$

3. $(3x-5)8 = \frac{1+2x}{3x-5} (3x-5)$

$x \neq \frac{5}{3}$

$$24x-40=1+2x$$

$$\begin{array}{r} -2x \\ -2x \end{array}$$

$$22x-40=1$$

$$\begin{array}{r} +40 \\ +40 \end{array}$$

$$22x=41$$

$\boxed{x=41/22}$

4. $6(y+1) \left(\frac{4}{3} - \frac{y}{y+1} = \frac{1}{2} \right) = \frac{26(y+1)4}{3} - \frac{6(y+1)y}{y+1} = \frac{38(y+1) \cdot 1}{2}$

$y \neq -1$

$$8(y+1) - 6y = 3(y+1)$$

$$8y+8-6y=3y+3$$

$$2y+8=3y+3$$

$$\begin{array}{r} -2y \\ -2y \end{array} \quad \begin{array}{r} -3 \\ -3 \end{array} \quad \begin{array}{r} 2y-3 \\ 2y-3 \end{array}$$

$\boxed{5=y}$

5. $4(d+2) \left(\frac{-3d}{4d+8} + 2 = \frac{5}{d+2} \right) = \frac{4(d+2)(-3d)}{4(d+2)} + \frac{4(d+2)2}{1} = \frac{4(d+2)5}{d+2}$

$d \neq -2$

$$-3d + 8(d+2) = 20$$

$$-3d + 8d + 16 = 20$$

$$5d + 16 = 20$$

$$\begin{array}{r} -16 \\ -16 \end{array} \quad \begin{array}{r} -16 \\ -16 \end{array}$$

$$5d = 4$$

$$\frac{5d}{5} = \frac{4}{5}$$

$\boxed{d = \frac{4}{5}}$

6. $\frac{-4}{n-2} = \frac{n}{3n-6}$

$n \neq 2$

$$n(n-2) = -4(3n-6)$$

$$n^2 - 2n = -12n + 24$$

$$\begin{array}{r} +12n \\ +12n \end{array} \quad \begin{array}{r} +12n \\ +12n \end{array}$$

$$n^2 + 10n = 24$$

$$\begin{array}{r} -24 \\ -24 \end{array} \quad \begin{array}{r} -24 \\ -24 \end{array}$$

$$n^2 + 10n - 24 = 0$$

$$(n+12)(n-2) = 0$$

$\boxed{n = -12, 2}$ *extraneous!*

7. $(r+2)(r+4) \left(\frac{1}{r+2} + \frac{r-1}{r^2+6r+8} = \frac{1}{r+4} \right) = \frac{(r+2)(r+4) \cdot 1}{r+2} + \frac{(r+2)(r+4)(r-1)}{(r+2)(r+4)} = \frac{(r+2)(r+4) \cdot 1}{r+4}$

$r \neq -2, -4$

$$r+4 + r-1 = r+2$$

$$2r+3 = r+2$$

$$\begin{array}{r} -r \\ -r \end{array} \quad \begin{array}{r} -3 \\ -3 \end{array} \quad \begin{array}{r} -r-3 \\ -r-3 \end{array}$$

$\boxed{r = -1}$

$$8. \quad \frac{1}{5w-5} = \frac{1}{w-3} + \frac{w+2}{5w^2-20w+15} = \frac{5(w-3)(w-1) \cdot 1}{5(w-1)} = \frac{5(w-3)(w-1) \cdot 1}{w-3} + \frac{5(w-3)(w-1)(w+2)}{5(w-3)(w-1)}$$

$$w \neq 1, 3$$

$$\begin{aligned} w-3 &= 5(w-1) + w+2 \\ w-3 &= 5w-5 + w+2 \\ w-3 &= 6w-3 \\ \underline{-w+3} & \quad \underline{-w+3} \end{aligned}$$

$$\begin{aligned} 0 &= 5w \\ \frac{0}{5} &= \frac{5w}{5} \\ 0 &= w \end{aligned}$$

$$9. \quad 2k^2 \left(\frac{1}{k^2} + \frac{k+3}{2k} = \frac{1}{2} \right) = \frac{2k^2 \cdot 1}{k^2} + \frac{2k^2(k+3)}{2k} = \frac{2k^2 \cdot 1}{2}$$

$$k \neq 0$$

$$\begin{aligned} 2 + k(k+3) &= k^2 \\ 2 + k^2 + 3k &= k^2 \\ \underline{-k^2} & \quad \underline{-k^2} \\ 2 + 3k &= 0 \\ \underline{-2} & \quad \underline{-2} \end{aligned}$$

$$\begin{aligned} 3k &= -2 \\ \frac{3k}{3} &= \frac{-2}{3} \\ k &= -\frac{2}{3} \end{aligned}$$

$$10. \quad \frac{9}{h^2-6h+9} = \frac{3h}{h^2-3h} = \frac{h(h-3)(h-3) \cdot 9}{(h-3)(h-3)} = \frac{h(h-3)(h-3) \cdot 3h}{h(h-3)}$$

$$h \neq 0, 3$$

$$\begin{aligned} 9h &= h(h-3) \cdot 3 \\ 9h &= (h^2-3h) \cdot 3 \\ 9h &= 3h^2-9h \\ \underline{-9h} & \quad \underline{-9h} \\ 0 &= 3h^2-18h \\ 0 &= 3h(h-6) \\ h &= 0, 6 \end{aligned}$$

$h = 0$ is extraneous, so $h = 6$ is the only solution

Multiple Choice

11. Which of the following values of x solves: $\frac{2}{x-3} = \frac{1}{x^2-2x-3}$?

(A) $x = -3, -\frac{1}{2}$

(B) $x = -\frac{1}{2}, 3$

(C) $x = -\frac{1}{2}$

(D) $x = 3$

(E) None of the above

$$(x-3)(x+1) \left[\frac{2}{x-3} = \frac{1}{(x-3)(x+1)} \right] \rightarrow 2(x+1) = -1$$

$$2x+2 = -1$$

$$2x = -3$$

$$x = -\frac{3}{2}$$

FREE RESPONSE

12. Given the graph of the polynomial $g(x)$.

a. Write a possible equation of $g(x)$.

$$g(x) = x(x+4)(x-5)^2$$

b. Describe the end behavior of $g(x)$.

$$x \rightarrow -\infty$$

$$g(x) \rightarrow \infty$$

$$x \rightarrow \infty$$

$$g(x) \rightarrow \infty$$

