

11.1 Sequences

PRACTICE

<table border="1"> <thead> <tr> <th>N</th> <th>F(n)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>8 $\times 1.5$</td> </tr> <tr> <td>2</td> <td>12 $\times 1.5$</td> </tr> <tr> <td>3</td> <td>18 $\times 1.5$</td> </tr> <tr> <td>4</td> <td>27 $\times 1.5$</td> </tr> </tbody> </table>	N	F(n)	1	8 $\times 1.5$	2	12 $\times 1.5$	3	18 $\times 1.5$	4	27 $\times 1.5$	<p>1) What is the explicit formula for this sequence?</p> $F(n) = 8(1.5)^{n-1}$	<p>2) What's the 14th term?</p> $F(14) = 8(1.5)^{14}$ $F(14) = 8(1.5)^{14}$ $F(14) = 1556.956055$	<p>3) What term has the value of 205.03125?</p> $205.03125 = \frac{8(1.5)^{n-1}}{8}$ $25.62890625 = 1.5^{n-1}$ $\log_{1.5} 25.62890625 = n-1$ $5 = n-1$
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<table border="1"> <thead> <tr> <th>N</th> <th>F(n)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>-247 $+18$</td> </tr> <tr> <td>2</td> <td>-229 $+18$</td> </tr> <tr> <td>3</td> <td>-211 $+18$</td> </tr> <tr> <td>4</td> <td>-193 $+18$</td> </tr> </tbody> </table>	N	F(n)	1	-247 $+18$	2	-229 $+18$	3	-211 $+18$	4	-193 $+18$	<p>4) What is the explicit formula for this sequence?</p> $f(n) = -247 + 18(n-1)$	<p>5) What's the 20th term?</p> $F(20) = -247 + 18(20-1)$ $= -247 + 18(19)$ $= -247 + 342$ $f(20) = 95$	<p>6) What term has the value of 599?</p> $599 = -247 + 18(n-1)$ $\frac{846}{18} = \frac{18(n-1)}{18}$ $47 = n-1$ $48 = n$
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<table border="1"> <thead> <tr> <th>N</th> <th>F(n)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2500 -205</td> </tr> <tr> <td>2</td> <td>2295 -205</td> </tr> <tr> <td>3</td> <td>2090 -205</td> </tr> <tr> <td>4</td> <td>1885 -205</td> </tr> </tbody> </table>	N	F(n)	1	2500 -205	2	2295 -205	3	2090 -205	4	1885 -205	<p>7) What is the explicit formula for this sequence?</p> $f(n) = 2500 - 205(n-1)$	<p>8) What's the 30th term?</p> $F(30) = 2500 - 205(30-1)$ $= 2500 - 205(29)$ $= 2500 - 5945$ $f(30) = -3445$	<p>9) What term has the value of -1600?</p> $-1600 = 2500 - 205(n-1)$ $-4100 = -205(n-1)$ $20 = n-1$ $21 = n$
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<table border="1"> <thead> <tr> <th>N</th> <th>F(n)</th> </tr> </thead> <tbody> <tr> <td>1</td> <td>2700 $\times .9$</td> </tr> <tr> <td>2</td> <td>2430 $\times .9$</td> </tr> <tr> <td>3</td> <td>2187 $\times .9$</td> </tr> <tr> <td>4</td> <td>1968.3 $\times .9$</td> </tr> </tbody> </table>	N	F(n)	1	2700 $\times .9$	2	2430 $\times .9$	3	2187 $\times .9$	4	1968.3 $\times .9$	<p>10) What is the explicit formula for this sequence?</p> $f(n) = 2700(.9)^{n-1}$	<p>11) What's the 7th term?</p> $F(7) = 2700(.9)^{7-1}$ $= 2700(.9)^6$ $f(7) = 1493.49907$	<p>12) What term has the value e of 1594.323</p> $1594.323 = \frac{2700(.9)^{n-1}}{2700}$ $.59049 = .9^{n-1}$ $\log_{.9} .59049 = n-1$ $5 = n-1$ $6 = n$
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Mr. Kelly is getting back into shape. The first week he decides each of his runs will last 10 minutes. Then each week after that he'll increase the runs by 4 minutes.

<p>13) What are the times his run will last for each of the first 5 weeks of his training?</p> <p>10, 14, 18, 22, 26</p>	<p>14) What is an explicit formula to model this situation?</p> $F(n) = 10 + 4(n-1)$	<p>15) How many minutes will his run be in 3 months?</p> <p>3 months = 12 weeks</p> $F(12) = 10 + 4(12-1)$ $= 10 + 44$ $= 54 \text{ minutes}$	<p>16) How many weeks will have been running if his daily runs last an hour?</p> $60 = 10 + 4(n-1)$ $50 = 4(n-1)$ $12.5 = n-1$ $13.5 = n$ 14 weeks
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Mr. Brust is also getting back into shape. The first week he decides each of his daily runs will also last 10 minutes. Then each week after that he decides that he'll increase his runs by 15%.

<p>17) What are the times his run will last for each of the first 5 weeks of his training?</p> <p>10, 11.5, 13.225, 15.20875, 17.4900625</p>	<p>18) What is an explicit formula to model this situation?</p> $F(n) = 10(1.15)^{n-1}$	<p>19) How many minutes will his run be in 3 months?</p> $F(12) = 10(1.15)^{12}$ $= 46.52 \text{ min}$	<p>20) How many weeks will have been running if his daily runs last at least an hour?</p> $60 = 10(1.15)^{n-1}$ $6 = 1.15^{n-1}$ $\log_{1.15} 6 = n-1$ $12.82 = n-1$ $13.8 = n$ 14 weeks
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Mr. Sullivan has decided to go back on the www.flippedmath.com website and correct all the mistakes that the Algebras have made over the years. He finds that there are 10,240 mistakes. WHOA!!!! He decides that each month he'll fix half of the remaining mistakes.

21) What's an explicit formula that models this situation?

$$F(n) = 10,240 \left(\frac{1}{2}\right)^{n-1}$$

22) How many mistakes will he fix in the 8th month?

$$\begin{aligned} F(8) &= 10,240 \left(\frac{1}{2}\right)^{8-1} \\ &= 10,240 \left(\frac{1}{2}\right)^7 \\ &= 80 \text{ mistakes} \end{aligned}$$

23) In what month will he only have to fix 5 mistakes?

$$\begin{aligned} 5 &= 10,240 \left(\frac{1}{2}\right)^{n-1} \\ \frac{5}{10,240} &= \frac{1}{2}^{n-1} \\ \log_x \left(\frac{5}{10,240}\right) &= n-1 \\ 11 &= n-1 \\ 12 \text{ months} \end{aligned}$$

Compounding Interest
(continuous compounding)

$$A = Pe^{rt}$$

Compounding Interest
(periodic compounding)

$$A = P \left(1 + \frac{r}{n}\right)^{nt}$$

% increase/decrease
per unit of time

$$f(x) = ab^x$$

24) You deposit \$800 in an account that pays 5.7% annual interest compounded continuously. How much will you have after 13 years?

$$A = 800(e^{0.057(13)})$$

$$A = \$1678.43$$

25) You deposit \$5 in an account that pays 24% annual interest compounded monthly. How much will you have after 20 years?

$$\begin{aligned} A &= 5 \left(1 + \frac{.24}{12}\right)^{12(20)} \\ &= 5 (1.02)^{240} \\ &= 5 (1.02)^{240} \end{aligned}$$

$$A = \$579.44$$

26) Mr. Brust invests \$5000 in an account that is compounded monthly at a rate of 9%. How many years will it take him to have \$23,000 in the account?

$$\frac{23,000}{5000} = \frac{5000 \left(1 + \frac{.09}{12}\right)^{12t}}{5000}$$

$$4.6 = (1.0075)^{12t}$$

$$\frac{\log_{1.0075} 4.6}{12} = \frac{12t}{12}$$

$$17 \text{ years} = t$$

27) Mr. Bean puts \$5000 in a mutual fund that increases in value by 11% each year. How many years will it take him to reach \$23,000 in his mutual fund?

$$23000 = 5000(1.11)^t$$

$$4.6 = 1.11^t$$

$$\log_{1.11} 4.6 = t$$

$$14.6 \text{ yrs} = t$$